$$\begin{array}{rcl} p \rightsquigarrow \mathsf{Mlistof}\, R\, L & \equiv & \mathsf{match}\, L\, \mathsf{with} \\ & & \mid \mathsf{nil} \;\Rightarrow\; [p = \mathsf{null}] \\ & & \mid X :: L' \;\Rightarrow\; \exists x p'.\quad p \rightsquigarrow \{\!|\mathrm{hd}{=}x;\ \mathrm{tl}{=}p'|\!\} \\ & & \qquad\qquad \star\ p' \rightsquigarrow \mathsf{Mlistof}\, R\, L' \\ & & \qquad\qquad \star\ x \rightsquigarrow R\, X \end{array}$$

Exercise: since $(p : \mathsf{loc})$ and $(x : \mathsf{Val})$ and $(X : A)$ for some A, what is the type of R? What is the type of Mlistof?

$$\begin{array}{rcl} p \rightsquigarrow \mathsf{MList}\, L & \equiv & \mathsf{match}\, L\, \mathsf{with} \\ & & \mid \mathsf{nil} \;\Rightarrow\; [p = \mathsf{null}] \\ & & \mid x :: L' \;\Rightarrow\; \exists p'.\quad p \rightsquigarrow \{\!|\mathrm{hd}{=}x;\ \mathrm{tl}{=}p'|\!\} \\ & & \qquad\qquad \star\ p' \rightsquigarrow \mathsf{MList}\, L' \end{array}$$

Exercise: define the identity representation predicate Id such that

$$p \rightsquigarrow \mathsf{Mlistof}\, \mathsf{Id}\, L \;=\; p \rightsquigarrow \mathsf{MList}\, L$$

Exercise: specify functions over queues using a higher-order representation predicate written $p \rightsquigarrow \mathsf{Queueof}\, R\, L$. Shorthand: just write "Q R" instead of "Queueof R".

$\{\qquad\qquad\}\ (\texttt{create()})\ \{\qquad\qquad\qquad\qquad\qquad\qquad\}$

$\{\qquad\qquad\qquad\qquad\qquad\}\ (\texttt{push}\, x\, p)\ \{\qquad\qquad\qquad\qquad\qquad\qquad\}$

$\{\qquad\qquad\qquad\qquad\qquad\}\ (\texttt{pop}\, p)\ \{\qquad\qquad\qquad\qquad\qquad\qquad\}$

$\{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\}\ (\texttt{concat}\, p\, p')$
$\{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\}$

Exercise: specify a function $\texttt{copy}\, f\, p$ that duplicables a mutable queue specified using Queueof, where f is a function to duplicate items.

$$\begin{array}{ll} & (\forall x X.\ \{\qquad\qquad\qquad\qquad\}\ (f\ x)\ \{\qquad\qquad\qquad\qquad\qquad\qquad\}) \\ \Rightarrow & \{p \rightsquigarrow \mathsf{Queueof}\,R\,L\} \\ & (\mathsf{copy}\ f\ p) \\ & \{\lambda p'.\ p \rightsquigarrow \mathsf{Queueof}\,R\,L \star p' \rightsquigarrow \mathsf{Queueof}\,R\,L\} \end{array}$$

$$\begin{array}{rcl} p \rightsquigarrow \mathsf{MCellof}\,R_1\,V_1\,R_2\,V_2 & \equiv & \exists v_1 v_2.\quad p \rightsquigarrow \{\!|\mathrm{hd}{=}v_1;\ \mathrm{tl}{=}v_2|\!\} \\ & & \star\ v_1 \rightsquigarrow R_1\,V_1 \\ & & \star\ v_2 \rightsquigarrow R_2\,V_2 \end{array}$$

Exercise: rewrite the specification of Mlistof using MCellof.

$$\begin{array}{rcl} p \rightsquigarrow \mathsf{Mlistof}\,R\,L & \equiv & \mathsf{match}\ L\ \mathsf{with} \\ & & \mid \mathsf{nil}\ \Rightarrow\ [p = \mathsf{null}] \\ & & \mid X :: L'\ \Rightarrow \end{array}$$

Exercise: rewrite the specification of Narytreeof using Nodeof.

$$\begin{array}{l} p \rightsquigarrow \mathsf{Narytreeof}\,R\,T \equiv \\ \quad \mathsf{match}\ T\ \mathsf{with} \\ \quad \mid \mathsf{Leaf}\ \Rightarrow\ [p = \mathsf{null}] \\ \quad \mid \mathsf{Node}\ X\ L\ \Rightarrow \end{array}$$

Exercise: complete the specification of Bagof using Nodeof. Hint: chunks are described by the predicate $p' \rightsquigarrow \mathsf{Chunkof}\,R\,E'$.

$$\begin{array}{l} p \rightsquigarrow \mathsf{Bagof}\,R\,T \equiv \\ \quad \mathsf{match}\ T\ \mathsf{with} \\ \quad \mid \mathsf{Empty}\ \Rightarrow\ [p = \mathsf{null}] \\ \quad \mid \mathsf{Layer}\ E'\ T'\ \Rightarrow \end{array}$$

Exercise: specify the function `miter`, using an invariant of the form $J\,K\,K'$, describing the state before and the state after the iteration.

$$\begin{array}{ll} \forall f p R L J. & (\forall x X\qquad\qquad\qquad.\ \{x \rightsquigarrow R\,X\qquad\qquad\qquad\} \\ & \qquad\qquad\qquad\qquad\ (f\ x) \\ & \qquad\qquad\qquad\qquad\ \{\lambda\_.\qquad\qquad\qquad\qquad\qquad\}\ ) \\ \Rightarrow & \{p \rightsquigarrow \mathsf{Mlistof}\,R\,L\ \star\qquad\qquad\qquad\} \\ & (\mathtt{miter}\ f\ p) \\ & \{\lambda\_.\qquad\qquad\qquad\qquad\qquad\} \end{array}$$

```
let incr_all p =
  miter (fun x -> incr x) p

let example_p =
  { hd = ref 5; tl = { hd = ref 3; tl = null } }
```

$$x \rightsquigarrow \mathsf{Ref}\, X \quad \equiv \quad x \mapsto X$$

Exercise: using the representation predicates Ref and Mlistof, specify the function `(fun x -> incr x)` and `incr_all`.

$\{\qquad\qquad\qquad\qquad\}$ `(incr x)` $\{\lambda\_.\qquad\qquad\qquad\qquad\qquad\qquad\}$

$\{\qquad\qquad\qquad\qquad\qquad\qquad\}$ `(incr_all` p`)` $\{\lambda\_.\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\}$

Exercise: Describe the state at the front of each lines (except 5 and 6). Explicit the instantiation of the existential in the invariant.

```
1   let r = ref 0
2   let s = ref n
```

```
3   let p = create_lock()
4
5   let concurrent_step () =
6     let () = acquire_lock p in
7     incr r;
8     decr s;
9     release_lock p
```

Exercise: state a conversion rule relating $p \rightsquigarrow \mathsf{Cellsof}\, R\, M$ with a predicate of the form $p \rightsquigarrow \mathsf{Cellsof}\, \mathsf{Id}\, M'$. Hint: $(R : A \to a \to \mathsf{Hprop})$ and $(M : \mathsf{map}\, \mathsf{int}\, A)$ and $(M' : \mathsf{map}\, \mathsf{int}\, a)$.

$p \rightsquigarrow \mathsf{Cellsof}\, R\, M \;=$