

**Exercise 1.** describe the frame process for in-place increment.

**Exercise 2.** specify the tree copy function.

**Exercise 3.** describe the frame process for tree copy.

**Exercise 4.** give small footprint specifications for array operations. How to derive the large footprint specifications from them?

{		}	{		}
	} (Array.get i p)				
{		}	{		}
	} (Array.set i p v)				
{		}	{		}
	} (Array.length p)				

**Exercise 5.** give a small-footprint specification for quicksort.

**Exercise 6.** For each heap implication below, say whether it is true or false.

1.  $(r \mapsto 3) \star (s \mapsto 4) \triangleright (s \mapsto 4) \star (r \mapsto 3)$
2.  $(r \mapsto 3) \triangleright (s \mapsto 4) \star (r \mapsto 3)$
3.  $(s \mapsto 4) \star (r \mapsto 3) \triangleright (r \mapsto 4)$
4.  $(s \mapsto 4) \star (r \mapsto 3) \triangleright (r \mapsto 3)$
5.  $[\text{False}] \star (r \mapsto 3) \triangleright (s \mapsto 4) \star (r \mapsto 4)$
6.  $(r \mapsto 4) \star (s \mapsto 3) \triangleright [\text{False}]$
7.  $(r \mapsto 4) \star (r \mapsto 3) \triangleright [\text{False}]$
8.  $(r \mapsto 3) \star (r \mapsto 3) \triangleright [\text{False}]$

**Exercise 7.** For each heap implication below, say whether it is true or false.

1.  $(r \mapsto 3) \triangleright \exists n. (r \mapsto n)$
2.  $\exists n. (r \mapsto n) \triangleright (r \mapsto 3)$
3.  $\exists n. (r \mapsto n) \star [n > 0] \triangleright \exists n. [n > 1] \star (r \mapsto (n - 1))$
4.  $(r \mapsto 3) \star (s \mapsto 3) \triangleright \exists n. (r \mapsto n) \star (s \mapsto n)$
5.  $\exists n. (r \mapsto n) \star [n > 0] \star [n < 0] \triangleright (r \mapsto n) \star (r \mapsto n)$

**Exercise 8.** show that GC-PRE is derivable from GC-POST and FRAME.

$$\frac{\{H\} t \{Q\}}{\{H \star GC\} t \{Q\}}$$

**Exercise 9.** give a specification of copy in terms of MtreeComplete; which rules are used to derive this specification?

**Exercise 10.** complete the rule for sequences.

$$\frac{\{ \quad \} t_1 \{ \quad \} \quad \{ \quad \} t_2 \{ \quad \}}{\{H\} (t_1 ; t_2) \{Q\}}$$

**Exercise 11.** complete the reasoning rule for let-bindings.

$$\frac{\{ \quad \} t_1 \{ \quad \} \quad \forall x. \{ \quad \} t_2 \{ \quad \}}{\{H\} (\text{let } x = t_1 \text{ in } t_2) \{Q\}}$$

**Exercise 12.** instantiate the rule for let-bindings on the following code.

$$\{r \mapsto 3\} (\text{let } a = !r \text{ in } a+1) \{Q\}$$

$$\begin{aligned} H &\equiv \\ Q &\equiv \\ Q' &\equiv \end{aligned}$$

**Exercise 13.** Reasoning rule for values:

$$\frac{\triangleright}{\{H\} v \{Q\}}$$