

**Exercise 1.** give heaps satisfying the following heap predicates

$\lceil \lceil \rceil$		$\lceil 0 = 1 \rceil$	
$\lceil 1 = 1 \rceil$		$\lceil 1 = 1 \rceil * \lceil 0 = 1 \rceil$	
$1 \mapsto 2$		$(1 \mapsto 2) * \lceil 1 = 1 \rceil$	
$(1 \mapsto 2) * (1 \mapsto 3)$		$(1 \mapsto 2) * (2 \mapsto 1)$	

**Exercise 2.**

1. state after `let r = ref 5 and s = ref 3 and t = r:`
2. state after subsequently executing `incr r:`
3. state after subsequently executing `incr t:`

**Exercise 3.** give heaps satisfying the following heap predicates

$\exists x. \lceil 1 \mapsto x \rceil$		$\exists x. (1 \mapsto x) * (2 \mapsto x)$	
$\exists x. \lceil x = x + 1 \rceil$		$\exists x. (x \mapsto x + 1) * (x + 1 \mapsto x)$	
$\exists x. 1 \mapsto x$		$\exists x. (x \mapsto 1) * (x \mapsto 2)$	
$\exists P. \lceil P \rceil$		$\exists H. H$	

**Exercise 4.** in-place list reversal

State before the loop:

State after the loop:

Loop invariant:

**Exercise 5.** length of mutable list using a while loop

State before the loop:

State after the loop:

Picture describing the state during the loop:

Try to state a loop invariant. What do you need?

**Exercise 6.** generalize MList to define  $p \rightsquigarrow \text{MlistSeg } q L$ , where  $L$  denotes the list of items in the list segment from  $p$  (inclusive) to  $q$  (exclusive):

$p \rightsquigarrow \text{MlistSeg } q L \equiv$

**Exercise 7.** length of mutable list using a while loop and MlistSeg

Loop invariant:  $\exists q, L_1, L_2. \dots$

Instantiate  $q, L_1, L_2$  before the loop:

Instantiate  $q, L_1, L_2$  after the loop:

**Exercise 8.** define the representation predicate  $p \rightsquigarrow \text{Queue } L$ .

**Exercise 9.** define the representation predicate  $p \rightsquigarrow \text{Mtree } T$ .

**Exercise 10.** define  $p \rightsquigarrow \text{MtreeDepth } n T$  by generalizing  $p \rightsquigarrow \text{Mtree } T$ .

**Exercise 11.** give an alternative definition of “ $p \rightsquigarrow \text{MtreeDepth } n T$ ”, this time by reusing the definition of  $p \rightsquigarrow \text{Mtree } T$  without modification.

**Exercise 12.** define a predicate  $p \rightsquigarrow \text{MtreeComplete } T$  for describing a mutable complete binary tree, of some unspecified depth.

**Exercise 13.** define a predicate  $p \rightsquigarrow \text{MsearchTree } E$  for describing a mutable binary search tree storing the set of elements  $E$ .

**Exercise 14.** specify the primitive operations on references.

`(ref v)`

`(!r)`

`(r := v)`

**Exercise 15.** Give specifications for:

`(Array.get i p)`

`(Array.set i p v)`

`(Array.length p)`

`(Array.create n v)`

**Exercise 16.** What is the *natural* specification of function `myref`? What is missing from our current interpretation of triple?